

Noether's Theorem: A Jet-Geometric Formulation via the Variational Bicomplex

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Abstract

In this short presentation, I outline a jet-geometric formulation of Noether's theorem using the variational bicomplex on the infinite jet bundle $J^\infty(E \rightarrow M)$. In this framework, variational symmetries, Euler–Lagrange equations, and Noether currents arise naturally from the horizontal–vertical decomposition and the cohomological structure encoded in Spencer operators and contact forms.

This formulation clarifies how conservation laws can be understood independently of explicit symmetry groups, revealing intrinsic geometric constraints of differential operators. I briefly indicate how these structures relate to symbolic and microlocal features—through the compatibility conditions of the Spencer complex and the behavior of principal symbols—suggesting a pathway toward microlocal interpretations of Noether-type identities.

Examples from geometric partial differential equations illustrate how the jet-bundle viewpoint helps organize conserved quantities, understand constraint propagation, and highlight stability phenomena. The aim is to give an accessible introduction showing how the variational bicomplex offers a unified geometric framework for conservation laws in variational and PDE systems.

References

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