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The chain recurrent set of flow of automorphisms on a solvable Lie group

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Abstract

Let $\varphi_t : \mathbb{K}^n \to \mathbb{K}^n$ be a flow of automorphisms on a metric space. An (ε, τ)-*chain* connecting a point x to y is the collection

$$\xi := \{n;\, x = x_0, x_1, \ldots, x_{n-1}, x_n = y;\, \tau_0, \tau_1, \ldots, \tau_{n-1}\},$$

such that $d(\varphi_{\tau_i}(x_i), x_{i+1}) < \varepsilon, \quad \text{and} \quad \tau \leq \tau_i, \quad \text{for } i = 0, \ldots, n-1.$

The set of points that can be reached from an initial point x through (ε, τ)-chains, for all $\varepsilon, \tau > 0$ in positive time, is called the *forward chain limit set* and is denoted by $\Omega(x)$. A point x is said to be *chain recurrent* if $x \in \Omega(x)$, and the set of chain recurrent points with respect to the flow φ_t is denoted by $\mathcal{R}_C(\varphi)$. This notion extends to Lie groups in terms of neighborhoods.

In the case of flows on $\mathbb{R}^n$, it was proved in [1] that

$$\mathcal{R}_C(\varphi) = L^0,$$

where L^0 is the generalized eigenspace corresponding to eigenvalues with real part equal to zero. On the other hand, in [2] it was shown that the chain recurrent set of a flow of automorphisms on a connected Lie group coincides with the central subgroup of the flow, that is,

$$\mathcal{R}_C(\varphi) = G^0.$$

In this presentation, we will go through the proof in the particular case of *decomposable solvable Lie groups* ($G = G^{+,-}G^0$), and we will present an explicit example in dimension 3.

References

[1] Fritz Colonius, Alexandre J. Santana and Eduardo C. Viscovini, Chain Controllability of Linear Control Systems. In: *SIAM Journal on Control and Optimization* 62.4 (2024), pp. 2387–2411. DOI: 10.1137/23M1626347.

[2] Adriano Da Silva and Jhon Eddy Pariapaza Mamani. The Chain Recurrent Set of Flow of Automorphisms on a Decomposable Lie Group. In: *Journal of Dynamical and Control Systems* 31.2 (May 2025), p. 17. DOI: 10.1007/s10883-025-09740-5.

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