Existence of Solutions to the $1-$Laplacian problem with a critical singular nonlinearity

Yino Cueva Carranza * [1], Marcos T. O. Pimenta [2], and Giovany M. Figueiredo [3]

^{1,2,3}*Universidade Estadual Paulista,Universidade de Brasília-Brasil*

Abstract

The purpose of this work is to find a positive solution to the following elliptic equation involving 1-Laplacian operator which combines a singular term and a critical one

$$\begin{cases} -\text{div}\left(\dfrac{Du}{|Du|}\right) &= \dfrac{\lambda}{u^\gamma} + |u|^{1^*-2}u & \text{in } \Omega \\ u &= 0 & \text{on } \partial\Omega, \end{cases} \tag{1}$$

where $\Omega \subset \mathbb{R}^N$ is a bounded open set having Lipschitz–continuous boundary, $N \geq 2$, $\lambda > 0$ and $0 < \gamma < 1$.

The proof of the existence of a solution to (1) follows the ideas developed in [1] and [3]. It is based on an approximation method in which the solution is obtained as the limit as $p \to 1^+$, of the family of solutions of the $p-$Laplacian associated problems. We point out that in the proof of this result, we have the additional difficulty brought by the critical nonlinearity. In order to deal with it, we shall use a result in [2], which is a version of the Lions' Concentration of Compactness Principle.

References

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*yino.cueva@unesp.br